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Abstract

A modified constitutive model for the behaviour of unsaturated expansive soils has been presented. The essential characteristic of this model is the consideration of a double structure in the soil. Basic swelling and shrinkage behaviour of active clay minerals is associated with the microstructure. On the other hand, typical irreversible phenomena associated with rearrangement of pore space can be properly described through the macrostructural part of the model. Interaction between above-mentioned two parts accounts for some typical features of the behaviour of expansive soils. The model has been cast in mathematical terms using concepts of the theory of hardening plasticity. In order to check the capabilities of the model, three suction controlled oedometer tests on a highly expansive double-structure clay have been analysed. The comparison has shown that the model is able to reproduce most observed experimental results with fidelity.

Introduction

The complicated issues of deformation behaviour of unsaturated expansive soils are basically the stress-path dependency of swelling deformation behaviour, the micro and macro deformation behaviour, and the coupling deformation behaviour between microstructural and macrostructural levels. On the basis of the limited suction controlled test results published up to date, relevant volumetric behaviour can be summarised in the following points: 1) The amount of swelling experienced by wetting has been shown to decrease with the intensity of the total mean (or vertical) stress applied to the sample (Brackley, 1975; Alonso et al., 1995); 2) Swelling strains developed upon first wetting exhibit irreversible as well as reversible components; 3) Swelling strains are strongly affected by the stress path; 4) Soils with a marked open structure, subjected to relatively high confining stresses, may experience a collapse (volume reduction) in the final stages of a wetting path (Alonso et al.,1995). Soils with a marked open structure, subjected to a low suction, may also experience a marked collapse (volume reduction) during a drying path (Alonso et al., 1995).

As we known, collapsible and moderately expansive soils have been successfully modelled by a theory (Alonso et al., 1990) which requires the independent consideration of two effective stress fields: $(\sigma_{ij} - u_a\delta_{ij})$, called net stress and $(u_a - u_w)\delta_{ij}$ or suction. Where σ_{ij} = total stress; u_a = pore air pressure; u_w = pore water pressure; δ_{ij} = Kronecker delta. This model requires the following parameters (Alonso et al., 1990):

- Parameters associated with the so-called Loading-Collapse (LC) yield curve: $\lambda(0)$, the compression index and κ, the swelling index measured from the virgin loading curves $e \sim \log p$ for saturated condition; κ_s, elastic swelling index against changes in suction; r, ratio of minimum value of the compression index (for high values of suction) to $\lambda(0)$; β, parameter which controls the rate of decrease of the compression index with suction. $(\lambda(s) = \lambda(0)[(1-r)\exp(-\beta s) + r])$.
- Parameters associated with changes in shear stress and the shear strength: G, shear modulus in the elastic domain; M, slope of the critical state line (related to the angle of internal friction); k, parameter which controls the rate of increase of apparent cohesion, p_s, with suction $p_s = k\,s$.

In order to describe and reproduce the complicated deformation behaviour of unsaturated expansive clays, conceptual constitutive models for the behaviour of unsaturated expansive soils need to be developed. In a recent paper, Gens and

Alonso (1992) have explored in a qualitative way the possibilities of coupling basic microstructural laws of expansion at particulate level with the above-mentioned unsaturated soil framework appropriate for the description of the macrostructure, where capillary effects are specially relevant. Further improvement in the mathematical formulation of the model needed to perform quantitative predictions has been made by Alonso et al. (1994) more recently. In this paper, using concepts of the theory of hardening plasticity and some hypothesis concerning the coupling between both levels of soil structure, a modified model capable of making predictions in a quantitative way has been developed, and the further comparison of the model predictions with new experimental observations has then be made accordingly to illustrate the reliability and capability of the model.

Microstructural Behaviour and Its Relationship with the Macrostructure

A reference model to understand the swelling behaviour of clays is the Gouy-Chapman double layer theory. On the basis of this theory, the volumetric deformation of assumed parallel stacking of clay particles is a function of ($p+s$), i.e. $e=f(p+s)$, where $p=\sigma_m-u_a$ is the net mean stress and $s=u_a-u_w$ is the matric suction. On the other hand, since the clay aggregates will likely remain saturated under suctions prevailing in geotechnical environments, the principle of effective stress will be valid within the aggregates. Accordingly, changes in microstructural volumetric strains, ε_{vm}, will depend on effective mean stress, ($p+s$), and possibly on other factors. In a (p, s) space, stress paths for which $p+s$ = constant imply zero microstructural volumetric change. For this reason the straight lines $p+s$ =constant have been called Neutral Lines (NL) (Gens & Alonso, 1992). The microstructure will undergo compression or swelling whenever $d(p+s)>0$ or $d(p+s)<0$, respectively.

However, in some cases, the double layer theory fails to predict observed behaviour, since the actual microfabric of clayey soils differs from the assumed parallelism of particles implied in the Gouy-Chapman double layer theory (Yong et al., 1984). According to the experimental test results (Alonso et al., 1995) on an active compacted clay with open macrostructure, changes in effective stress are more effective in inducing expansion than changes in suction. In a (p, s) space, the Neutral Lines ($\tilde{NL}$) should therefore be modified as the following simply equation (i.e. $\tilde{NL}$ line in Fig. 1):

$$s+\eta_m p = constant \tag{1}$$

Where η_m is a parameter determined by relevant laboratory test results. The Gouy-Chapman theory is a special case when $\eta_m=1$ in Eq. (1). A relationship between ε_{vm} and $s+\eta_m p$ can be specified in the model as: $\varepsilon_{vm}=f_m(s+\eta_m p)$. It is likely that exponential type of relationships is suitable for the function f_m (Alonso et al., 1994).

Swelling experiments in which suction reversals are induced (Pousada, 1984) indicate that a large proportion of the first swell is not recovered when suction is again increased. This irreversible macrostructural swelling strain will be named ε^p_{vM}. The second postulate for the modified model refers to the relationship between ε_{vm} and ε^p_{vM}. It was assumed that this relationship depends on the applied net mean stress and more specifically on the ratio p/p_0 in such a way that it becomes zero when $p=p_0$. The rationale behind this assumption is that $p=p_0$ corresponds to an open macrostructure susceptible of collapsing. Some interesting experimental results obtained by Gehling (1994) shows that microstructural

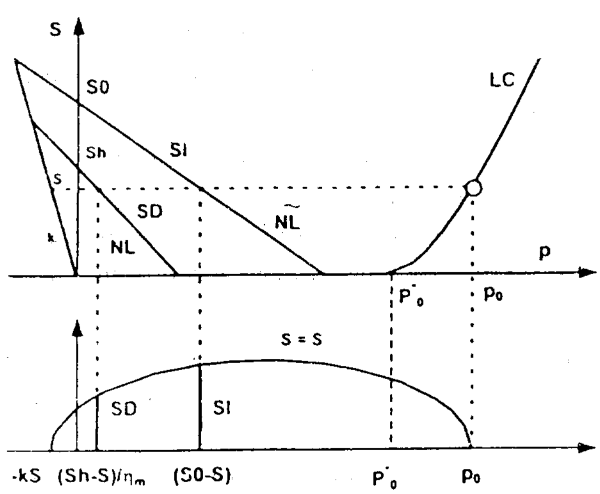

Fig. 1 Yield Surfaces in space (p, q, s).

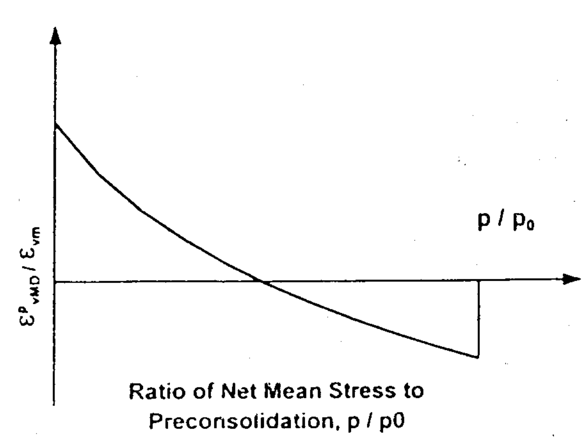

Fig. 2 Variation of macrostructural plastic volumtric strain with applied net stress

swelling volumetric strains upon the first wetting result in the marked macrostructural collapse when the applied net mean stress p closes to the preconsolidation net mean stress p_0. Decreasing values of p / p_0 imply denser packing and therefore the ratio $\varepsilon^p_{vM} / \varepsilon_{vm}$ should increase as p / p_0 decreases. Relationships of these two kinds have been plotted in Fig. 2.

Experimental test results (Alonso et al., 1995) on an active compacted clay with open macrostructure show that the marked irreversible macrostructural volumetric deformations of the testing samples occur along drying paths. In the testing samples with marked open macrostructures subjected to low suction, the principle of effective stress may be valid. Other experiments in which drying-wetting cycles are carried out showed plastic compression of the soil (Yong, Japp and How, 1971; Josa et al., 1987). As a consequence, the principle of effective stress will be adopted probably to reproduce the marked plastic compression of the soil upon drying. Another Neutral Line, $\tilde{NL}$:

$$s + p = constant \tag{2}$$

can be adopted as a yield locus in drying paths. In drying paths, the Neutral Line $\tilde{NL}$ may therefore be interpreted as a yield locus because, when it is reached, plastic shrinkage strains are induced. In next section these ideas will be formulated in mathematical terms.

Mathematical Formulation

1. Isotropic Conditions (p, s) Plane

The Neutral Lines, NL and $\tilde{NL}$ (Fig. 1) are associated with swelling volumetric deformations of the microstructure and shrinkage volumetric deformations of the macrostructure, respectively. According to the previous explanation, a given value of $(s + \eta_m p)$ or (s + p) corresponds to a particular position of line NL or $\tilde{NL}$. It is proposed that beyond a given value of $(s + \eta_m p)$ or (s + p) either in suction decreasing paths (swelling) or suction increasing paths (shrinkage) irreversible macro-structural deformations are induced. They will be called SI (for irreversible shrinkage deformations) and SD (for irreversible swelling). SI and SD stand for suction increase and suction decrease, respectively and have been plotted in Fig. 1. SI and SD act therefore as yield surfaces.

2. Elastic Deformations

(1) Macro-structural elastic deformations associated with changes in p. This is a 'Mechanical' effect described by

$$d\varepsilon^e_{vMp} = -\frac{\kappa}{v}\frac{dp}{p} \tag{3}$$

Where v is the specific void ratio ($v = 1 + e$).

(2) Macro-structural elastic deformations associated with changes in s. They are a result of changes in capillary stresses and have a mechanical effect:

$$d\varepsilon^e_{vMs} = -\frac{\kappa_s}{v}\frac{ds}{s + u_{atm}} \tag{4}$$

Where the constant u_{atm} (atmospheric pressure) has been introduced to avoid indeterminancies when $s = 0$.

(3) Microstructural elastic deformations associated with changes in $(s + \eta_m p)$. They correspond to the compression or expansion of clay aggregates:

$$d\varepsilon^e_{vm} = -\alpha_m \beta_m \exp\left[-\alpha_m (s + \eta_m p)\right] d(s + \eta_m p) \tag{5}$$

Where α_m, β_m *and* η_m are three parameters.

3. Plastic Deformation in Swelling and Shrinkage Paths

It is proposed by Alonso et al. (1994) that the amount of plastic strains induced when the stress path crosses yield loci SD or SI is proportional to the associated microstructural elastic strain (5). Therefore, a change in dp or ds leads to the following plastic deformations:

$$SD \text{ yield locus: } d\varepsilon^p_{vMD} = d\varepsilon^e_{vm} f_D(p, p_0) \tag{6}$$

$$SI \text{ yield locus: } d\varepsilon^p_{vMI} = d\varepsilon^e_{vm} f_I(p, p_0) \tag{7}$$

According to the recent research performed by Alonso et al. (1994), it seems appropriate that both functions exhibit increasing values with the increase of the distance (or difference) between p and p_0 and become zero when $p = p_0$

The following functions, i.e. $f_D = t_D(1-p/p_0)^{nD}$ and $f_I = t_I(1-p/p_0)^{nI}$, were proposed by Alonso et al. (1994) to satisfy the preceding criteria and to offer a convenient flexibility. t_D, t_I, n_D and n_I are model parameters. However, according to experimental results obtained by Gehling (1994), the above-mentioned function f_D can be further modified in a simply way as

$$\tilde{f}_D(p, p_0) = t_D\left(1 - \frac{p}{p_0}\right)^{n_D} - p_D\left(\frac{p}{p_0}\right)^{n_D} \tag{8}$$

On the basis of the more recent laboratory test results on an active compacted expansive clay (Alonso et al., 1995), it seems that the above-mentioned f_I is far off the possibility of describing the shrinkage volumetric strains in drying path. In accordance with a previous assumption that indicates the principles of effective stress may be valid in unsaturated soils with marked open macrostructures subjected to low suction, the equation (7) may be modified in a simply way as

$$SI \text{ yield locus} : d\varepsilon^{p}_{vMI} = -\frac{\lambda(s) - \kappa(s)}{v}\ \xi\ \frac{1}{s+p} d(s+p) \tag{9}$$

Where $\kappa(s)$ may be proposed in the same type of function by analogy with $\lambda(s)$ as: $\kappa(s) = \kappa(0)[(1-r)\exp(-\beta s) + r]$; ξ is a correct parameter for revising the discrepancy due to this assumption.

4. Hardening Laws

The mathematical equations of yield loci SD and SI defined previously are simply as

$$SD : F_D = s + \eta_m p - s_h = 0 \tag{10}$$

$$SI : F_I = s + p - s_0 = 0 \tag{11}$$

Where s_h and s_0 are stress parameters. A physical interpretation for s_h or s_0 (Fig. 1) is to identify them as the suction beyond which irreversible swelling or shrinkage strains are induced in wetting or drying paths under zero mean confining stress. s_h and s_0 are selected as hardening parameters which control the position of SD and SI and are supposed to depend on the accumulated plastic volumetric strain. Available experimental results are too scarce to provide sufficient evidence to accept that the position of loci SD and SI is related to each other. According to this fact, in this paper we give up an effort on consideration of the coupling effects or the interaction between the position of loci SD and SI.

It may be an important task to determine the initial position of loci SI and SD. It may be determined from laboratory suction controlled test results. The values depend not only on initial compacted water content, the first wetting path, but also on the open extent of macrostructure in unsaturated soils. However, for the initial position of loci SD ($s_h = s_{hi}$), it may be not ease to be determined from laboratory suction controlled test results for unsaturated soils with high initial suction or low initial water content. The parameter s_{hi} may therefore be approximated after a trial and error procedure using the measured results in laboratory. In a recent paper, Alonso et al. (1994) postulated that yield surface LC, whose original hardening law (Alonso et al., 1990) was given by $dp_0^* / p_0^* = d\varepsilon_v^p / [\lambda(0) - \kappa(0)]$, is now affected by irreversible wetting and drying phenomena and further modified as

$$\frac{dp_0^*}{p_0^*} = \frac{d\varepsilon_v^p + d\varepsilon_{vM}^p}{\lambda(0) - \kappa(0)} \tag{12}$$

Where p_0^* is the preconsolidation stress for saturated conditions and $d\varepsilon^p_{vM} = d\varepsilon^p_{vMD} + d\varepsilon^p_{vMI}$. Eq. (12) defines an interaction between yield loci LC and SI & SD.

5. Triaxial Stress States (p,q,s) Space

Extension to triaxial stress states follows the criteria established for the original model for unsaturated soils (macrostructure). The major axis of the yield ellipses extends from the apparent cohesion $p_s = -k\,s$ to the current net isotropic yield stress, p_0, and its equation is

$$q^2 - M^2(p + p_s)(p_0 - p) = 0 \tag{13}$$

The yield lines SI and SD can be assumed to extend into the region $q > 0$ by means of planes parallel to the q - axis so that Eqs. (10) and (11) remain unchanged.

Comparison of Model Predictions with Experimental Test Results

Details of the tests performed on unsaturated expansive Boom clay are given elsewhere (Alonso et al., 1995) and only a brief summary is given here. The soil used in the tests was a highly active expansive clay having the following characteristics: $w_L = 55.9\%$; $I_p = 26.7\%$; dry density γ_d: 19-21$kN / m3$; clay fraction: kaolinite 20%, illite 30%, montmorillonite 10%. The Boom clay formation is a tertiary marine sediment at depth of between -180 to -280m and situated at Mol, Belgium.

As we known, the macroscopic behaviour of unsaturated expansive clays can be studied by means of the distinction within a microstructural level where the basic swelling of the active minerals takes place and a macrostructural level responsible for major structural rearrangements. To illustrate the influence of each level and how alternative to the complex fabric of natural clays ideal samples have been tested in the laboratory.

These ideal samples have been made by pellets of clay powder with average size of 2 mm and dry density of 20 $kN / m3$ compacted into larger specimens of overall dry densities of 13 $kN / m3$, 14 $kN / m3$ and 15 $kN / m3$ respectively. The pellets simulate the particle arrangements observed in clay soils. Two kinds of test methods have been carried out, i.e. conventional oedometer and suction controlled oedometer test. Among the different results reported (Alonso et al., 1995), only three interesting suction controlled tests corresponding to the stress paths shown in Fig. 3a have been selected for comparison with the model developed in this paper.

Upon low overburden pressure, the sample with an overall dry density of 14 $kN / m3$ exhibits the swelling behaviour along the stress path of suction decreasing during the first wetting. But the sample exhibits the collapse behaviour along the same stress path during the first wetting under higher pressure. However, under a token pressure between low pressure and higher pressure, the sample shows the swelling behaviour first and then the collapse behaviour along the same stress path during the first wetting. The samples used in the tests were statically compacted at a water content 3%. After being compacted, therefore, the sample was set in suction controlled oedometer to follow a given stress path (see Fig. 3a), but to keep a high suction unchanged is impossible for the given oedometer. Therefore, the initial deformation induced by the overburden pressure under a high constant initial suction can be corrected by the results of conventional oedometer tests with the same conditions.

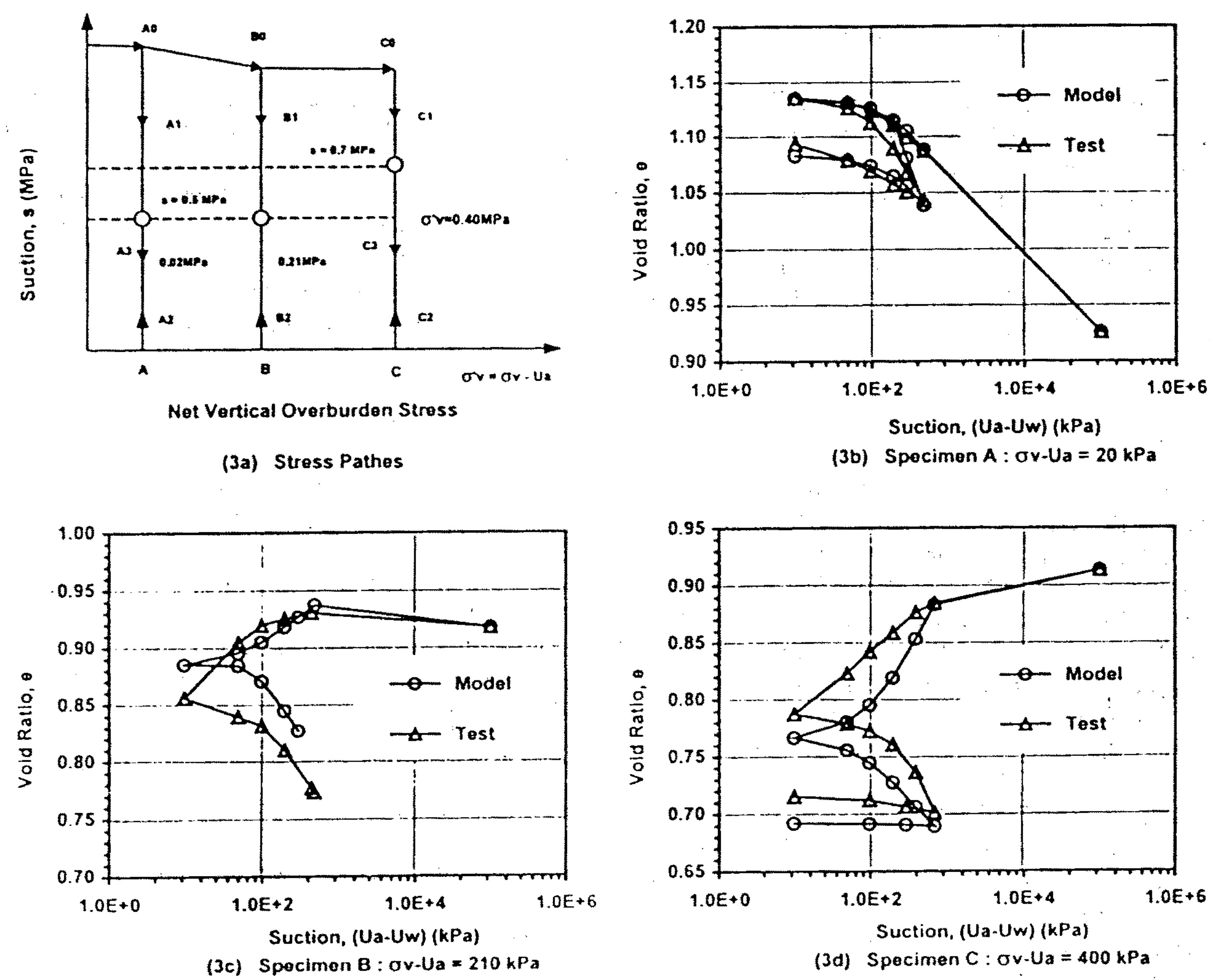

Fig. 3 Stress Paths and Comparison of Model Predictions with Experimental Test Results

The model parameters have been determined in the following way. The values of $\lambda(0)$, $\kappa(0)$ and $p_0^{\bullet}$ are obtained from the $e \sim \log p$ curves with saturation condition, i.e. $\lambda(0) = 0.158$, $\kappa(0) = 0.033$ and $p_0^{\bullet} = 0.11$ *MPa*, respectively. The initial yield values of the yielding loci of SI and SD, were found in the tests $s_{0i} = 0.2$ *MPa* and $s_{hi} = 0.7$ *MPa*, respectively. The values $p^C = 0.0934$ *MPa*, r = 0.12, $\beta = 1.5\ MPa^{-1}$, $\kappa_s = 0.0005$ were selected by a trial and error procedure using the measured results for the sample at an overburden pressure 0.4 *MPa*. The values $\alpha_m = 0.5\ MPa^{-1}$, $\beta_m = 0.1275$, $\eta_m = 27.4$, $t_d = 0.3$, $p_d = 0.3$, $n_d = 10$ and $\xi = 0.667$ were selected also by a trial and error procedure using the measured results for the samples at an overburden pressure 0.02 *MPa*. K_0=0.5, K=0.8 and M =1.2 were selected by referring other research work (Alonso et al., 1990).

The comparison between measured results and computed values for the above-mentioned three tests is given in Figures 3b, 3c and 3d. The comparison is quite acceptable. The model seems to capture the fundamental trends of behaviour and quantitative agreement is also encouraging. Under low overburden pressure and along first wetting path, the model predicts only reversible elastic swelling deformation before the SD line hits $s_{hi} = 0.7$ *MPa*, but together with irreversible plastic swelling deformation which drives the LC line from left to right when the SD line is below $s_{hi} = 0.7$ *MPa*. Subsequently, along first drying path the model demonstrates recovered elastic shrinkage deformation before the SI line exceeds $s_{0i} = 0.2$ *MPa* and together with obvious irreversible plastic shrinkage deformation that leads the LC line from left to right when the SD line is over $s_{0i} = 0.2$ *MPa*. Under higher overburden pressure, the model exhibits marked collapse behaviour along first wetting path, since the LC line was hit and then drove from left to right under high suction condition. Again, under a token overburden pressure between low pressure and higher pressure, the model is able to simulate the swelling behaviour first and then the collapse behaviour during the first wetting path. Since the model predicts reversible elastic and irreversible plastic swelling deformation before the LC line was reached under high suction condition and subsequently simulates irreversible plastic collapse deformation when the LC line was hit.

Concluding Remarks

A modified constitutive model for the behaviour of unsaturated expansive soils has been presented. The essential characteristic of this model is the consideration of a double structure in the soil. Basic swelling and shrinkage behaviour of active clay minerals is associated with the microstructure. On the other hand, typical irreversible phenomena associated with rearrangement of pore space can be properly described through the macrostructural part of the model. Interaction between above-mentioned two parts accounts for some typical features of the behaviour of expansive soils. The model has been cast in mathematical terms using concepts of the theory of hardening plasticity. In order to check the capabilities of the model, three suction controlled oedometer tests on a highly expansive double-structure clay have been analysed. The comparison has shown that the model is able to reproduce most observed experimental results with fidelity.

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