COUPLED MODEL FOR HEAT, MOISTURE, AIR FLOW AND DEFORMATION PROBLEMS IN UNSATURATED SOILS

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Coupled Model for Heat, Moisture, Air Flow, and Deformation Problems in Unsaturated Soils

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Abstract: Through extensive review and study on the different aspects of coupled processes in unsaturated soil, a general three-dimensional mathematical model for coupled heat, moisture, air flow, and deformation problems in unsaturated soils is proposed in a consistent and unified manner. In the proposed model, both pore-water and air transfers are assumed to be governed by the generalized Darcy's law and pore-vapor transfer is considered to occur due to two effects: first, under molecular diffusion and, second, as part of the bulk flow of the pore-air. The pore-vapor transfer due to the molecular diffusion is described using Fick's law. Heat transfer is formulated to include the effects of conduction, convection, and latent heat of vaporization. An elastoplastic framework is used to describe the deformation behavior of the unsaturated soil structure where linear elastic and nonlinear elastic models are two special cases. In particular, a generalized nonlinear constitutive model proposed earlier by the first writer is introduced and incorporated into the framework to predict the soil deformation with modification to account for the effect of temperature changes on deformation. The determination of the soil parameters involved in the coupled model is carefully discussed. Fully coupled, nonlinear differential equations are established and then solved by using a Galerkin weighted residual approach in space domain and an implicit integrating scheme in time domain. Finally, a robust new three-dimensional finite-element numerical computer program, C-HWAM-3D, is developed to incorporate the proposed mathematical model for analyzing the transient coupled flows of heat, moisture, and air, and the stress and strain in unsaturated soils.

INTRODUCTION

The behavior of unsaturated soils has become an important topic of modern soil mechanics. A lack of proper understanding and the rapidly increasing demand for the application of unsaturated soil mechanics to practical engineering have resulted in a significant development in this area around the world. In arid or semiarid parts of the world, many geotechnical engineering problems are associated with unsaturated soils. In western Europe, southwestern China, Canada, the United States, and many other regions, climatic changes could induce unexpected shrinkage and swelling movements that result in extensive damage to light buildings founded on swelling clays. In tropical areas, numerous slope failures occur after a prolonged period of rainfall. These landslides result in the loss of property and even lives. Fills, embankments, and earth dams are typical geotechnical problems related to unsaturated compacted soils. The stability of canals over swelling clays and/or collapsible soils is much related to water movements and mechanical behavior of unsaturated soils. The analysis of heat and mass flows in porous media is another important topic in many practical engineering problems. These problems include the accumulation of moisture under pavements, geothermal energy use, thermal enhancement of oil recovery, and the disposal of hazardous wastes such as radioactive wastes. Heat and mass flows in porous media are also related to hydrology, geology, meteorology, and agricultural problems.

The design of nuclear waste disposal schemes is currently under active consideration by a number of atomic energy authorities. A number of proposed schemes for radioactive waste disposal in crystalline rock or clay formations involve placing a canister containing the waste in a borehole drilled from underground galleries. The space between the borehole wall and canister is filled with compacted clays. Usually, highly compacted clays with highly expansive properties are selected as the most suitable material for constructing this type of engineered barriers. Once the sealing clay and canister are installed, a complex process will take place. Heat produced by the waste will cause drying and shrinkage of the clay close to the canister. Simultaneously, the initially unsaturated clay undergoes a hydration process by drawing water from the surrounding host material. Hence, a number of interrelated phenomena involving thermal, hydraulic, and mechanical aspects occur in the region around the waste that can lead to permanent changes in the final state of the engineered barrier. Because of the importance and the complexity of this problem, an improved understanding of the thermohydromechanical processes involved is therefore required. Considerable attention has to be given to the laboratory testing of backfill materials, in-situ testing and also the analyses of the transient phase of this barrier under a wide variety of conditions. An analysis of this problem requires the consideration of heat transfer, moisture migration, air transfer, and stress-strain behavior of unsaturated soil. The processes are interrelated and the approach requires coupling of these phenomena. This paper attempts to formulate the theoretical basis of this coupled analysis.

Since the 1950s, development in hydrology and agriculture research has resulted in many contributions on the analysis of nonisothermal moisture flows in unsaturated soils without consideration on the deformation of soil structure (Philip and de Vries 1957; Sophocleous 1979; Milly 1982, 1984; Celia et al. 1990; Chanzy and Bruckler 1993; Wilson et al. 1994). Among these contributions, the theory of Philip and de Vries (1957) provides the most comprehensive basis for the prediction of heat and moisture transfer in unsaturated soils. Philip and de Vries (1957) assumed liquid flow in response to a volumetric water content gradient. However, fundamentally the flow of liquid water occurs in response to a hydraulic-head gradient. The volumetric water content based formulation is applicable only to the analysis of homogeneous and isotropic systems. Geotechnical engineers are commonly required to analyze multilayered, anisotropic systems. Because of this limitation,

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Sophocleous (1979) and Milly (1982, 1984) used a matric suction-head based formulation to modify the volumetric water content based formulation (Philip and de Vries 1957). However, both the volumetric water content based formulation and the matric suction-head based formulation are not suitable for analyses of heat and moisture transfer in saturated soils and also in deformable unsaturated soils. In saturated soils, the volumetric water content gradient and the matric suction-head gradient could be zero and yet water could still flow through the soils. In deformable unsaturated soils, both the volumetric water content based formulation and the matric suction-head based formulation fail to take account of the effects of the changes in the pore-air pressure due to temperature and loading, and of consolidation on the liquid flow that occurs in response to a hydraulic-head gradient. This paper assumes a hydraulic-head formulation for liquid flow, i.e., pore-water transfer is assumed to be governed by the generalized Darcy's law.

In recent years, with growing effort in this field and the development of powerful numerical techniques, much progress on heat and mass transfer in deformable unsaturated soils has been achieved (Geraminegad and Saxena 1986; Pollock 1986; Ewen and Thomas 1989; Thomas and King 1991; Thomas et al. 1995; Thomas and He 1995; Olivella 1995). Using a matric suction-head based formulation for the prediction of moisture transfer in unsaturated soil, Geraminegad and Saxena (1986) developed a mathematical model that incorporated soil volumetric deformation due to a change in suction and pore-air pressure, but did not incorporate soil deformation due to external loading. Because of the aforementioned limitations, the model proposed by Geraminegad and Saxena (1986) is not applicable for saturated porous media and deformable unsaturated porous media. Thomas and He (1995) have recently analyzed the coupled heat, moisture, and air transfer in a deformable unsaturated soil. The work has extended previous analyses of heat and moisture transfer in unsaturated soil to take into account the deformation and stress-strain behavior of the soil. The progress has been achieved through the use of elasticity theory coupled with the so-called state surface approach to relate volumetric strain not only to stress, but also to suction and temperature changes. More recently, Olivella (1995) has also proposed a formulation for the coupled analysis of thermohydromechanical problems in saline media. The work ranged from the establishment of the mass, momentum, and energy balance equations to its numerical solution. The main achievement in his research is the consideration of the solubility of salt in water and the creep volumetric deformation of salt within the elasticity theoretical framework. However, a more general mathematical model for describing coupled heat, moisture, and air flow with consideration of nonlinear elastic or elastoplastic deformation behavior of the unsaturated soil structure is therefore needed for theoretical research and practical usage.

Analytical solutions to the aforementioned existing mathematical models do not exist except for few special cases. Numerical methods such as the finite-element method or the finite-difference method are commonly used to develop computer programs for modeling practical problems. All practical engineering problems, in general, are three-dimensional (3D) and normally require spatial numerical analyses, except for some cases that can be simplified to one-dimensional or two-dimensional problems. However, to the writers' knowledge, no literature has referred to 3D modeling for coupled thermohydromechanical problems in unsaturated soils. Therefore, a general 3D numerical model, i.e., numerical computer program, for coupled heat, moisture, air flow, and deformation problems in unsaturated soils is urgently required for theoretical research and practical usage.

Through extensive study and review of the recent developments made on different aspects of coupled processes in unsaturated soil, this paper proposes a more general mathematical model that incorporates the different aspects of unsaturated soil behavior in a consistent and unified manner. In the model, pore-water and pore-air transfers are assumed to be governed by Darcy's law, and pore-vapor transfer is considered to occur due to two effects: molecular diffusion and as part of the bulk flow of the pore-air. The pore-vapor transfer due to molecular diffusion is described using Fick's law. Heat transfer is formulated to include the effects of conduction, convection, and latent heat of vaporization. An elastoplastic framework is used to describe the deformation behavior of the unsaturated soil structure where linear elastic and nonlinear elastic models become two special cases. In particular, a generalized nonlinear constitutive model proposed by Yang and Shen (1992) is incorporated into the framework to predict the soil deformation with modification to account for the effect of temperature changes on deformation. The determination of the soil parameters involved in the coupled model is carefully discussed. The fully coupled, nonlinear partial differential equations are established and then solved using a Galerkin weighted residual approach in space domain and an implicit integrating scheme in time domain. Finally, a robust new 3D finite-element numerical computer program, C-HWAM-3D, is developed to incorporate the proposed mathematical model for analyzing the transient coupled flows of heat, moisture, and air, and the stress and strain in unsaturated soils.

BASIC EQUATIONS

In an unsaturated soil, the three basic phases are solid, liquid, and air. The solid phase is assumed to be a continuous medium, that is, soil structure with nonlinear elastic deformation behavior or elastoplastic deformation behavior. The deformation of the soil structure is assumed to be small. The liquid phase is considered to be continuous pore-water containing dissolved "dry" air. The air phase is composed of two continuous subphases: vapor phase and dry air phase. The so-called dry air is the air that does not contain water vapor. Adopting the sign convention for stress and strain in soil mechanics, both compressive stress and strain are defined as positive. Based on the three basic laws of nature, i.e., the law of force equilibrium, the law of mass conservation, and the law of heat conservation, the governing partial differential equations for the problems in unsaturated soil can be derived.

Governing Equations for Soil Structure

The static stress equilibrium equation can be written as

$$\mathbf{L}_\sigma \boldsymbol{\sigma} + \mathbf{b} = 0 \tag{1}$$

where $\mathbf{L}_\sigma = [l_{ij}^\sigma]$, $(i = 1 \sim 3; j = 1 \sim 6)$; $l_{11}^\sigma = l_{24}^\sigma = l_{36}^\sigma = \partial/\partial x$; $l_{14}^\sigma = l_{22}^\sigma = l_{35}^\sigma = \partial/\partial y$; $l_{16}^\sigma = l_{25}^\sigma = l_{33}^\sigma = \partial/\partial z$; the rest of the elements in $\mathbf{L}_\sigma$ = zero; and $\boldsymbol{\sigma}$ and $\mathbf{b}$ = total stress vector and body force vector, respectively. The deformation equation in incremental form is

$$\Delta\boldsymbol{\varepsilon} = -\mathbf{L}_\varepsilon \Delta\mathbf{U} \tag{2}$$

where $\mathbf{L}_\varepsilon = \mathbf{L}_\sigma^T = [l_{ij}^\varepsilon]$, $(i = 1, 2, 3; j = 1 \sim 6)$; $\Delta\boldsymbol{\varepsilon}$ and $\Delta\mathbf{U}$ = vectors of strain increment and displacement increment, respectively; and $\mathbf{L}_\sigma^T$ = transposed matrix of $\mathbf{L}_\sigma$. The stress-strain relationship used in this paper is given in incremental form as

$$\begin{aligned}\Delta\boldsymbol{\sigma}^* = {} & [\mathbf{D}_e - \mathbf{D}_e(\partial Q/\partial\boldsymbol{\sigma}^*)(\partial F/\partial\boldsymbol{\sigma}^*)^t\mathbf{D}_e/(H - H_{cr})]\Delta\boldsymbol{\varepsilon} \\ & - \{\mathbf{D}_e(\mathbf{h}_e + \mathbf{h}_p) + [-(\partial F/\partial\boldsymbol{\sigma}^*)^t\mathbf{D}_e(\mathbf{h}_e + \mathbf{h}_p) + \partial F/\partial s] \\ & \cdot\mathbf{D}_e(\partial Q/\partial\boldsymbol{\sigma}^*)/(H - H_{cr})\}\Delta s + [\mathbf{D}_e - \mathbf{D}_e(\partial Q/\partial\boldsymbol{\sigma}^*) \\ & (\partial F/\partial\boldsymbol{\sigma}^*)^t\mathbf{D}_e/(H - H_{cr})]\boldsymbol{\beta}^T\mathbf{I}\Delta T\end{aligned} \tag{3}$$

where $\mathbf{D}_e$ = elastic matrix; $\mathbf{h}_e$ and $\mathbf{h}_p$ = elastic and plastic matric suction coefficient vectors, respectively; $\boldsymbol{\sigma}^*$ = net normal stress vector (i.e., $\boldsymbol{\sigma} - \mathbf{I}u_a$); $\mathbf{I} = [1, 1, 1, 0, 0, 0]^T$; H = plastic modulus, i.e., $-(\partial F/\partial \boldsymbol{\chi})(\partial \boldsymbol{\chi}/\partial \boldsymbol{\varepsilon}^p)(\partial Q/\partial \boldsymbol{\sigma}^*)$; H_{cr} = critical plastic modulus, i.e., $-(\partial Q/\partial \boldsymbol{\sigma}^*)^t \mathbf{D}_e(\partial Q/\partial \boldsymbol{\sigma}^*)$; $\boldsymbol{\chi}$ = vector of hardening parameters that depend on the total plastic strains $\boldsymbol{\varepsilon}^p$; $\boldsymbol{\varepsilon}^p$ = vector of plastic strains; F and Q = yield function and the plastic potential function of the unsaturated soils; the parameter β^T = linear thermal expansion coefficient of the soil medium; $s = (u_a - u_w)$ = matric suction; the pore-air pressure, $u_a = u_{ad}$ plus u_v; and u_w, u_v, u_{ad}, and T = pore-water pressure, the partial pressure of the water vapor in the pore spaces, the pore-dry air pressure, and the absolute temperature (K), respectively. The yield function F in (3) is given as (Alonso et al. 1990)

$$F = p + \frac{q^2}{M^2(p_s + p)} - p_0 = 0 \tag{4}$$

where p = mean normal stress, i.e., $(\sigma_x^* + \sigma_x^* + \sigma_x^*)/3$; q = generalized shear stress, i.e., $\sqrt{6}\tau_\sigma/2$; and τ_σ = π-plane shear stress given as

$$\tau_\sigma = \frac{1}{\sqrt{3}} [(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)]^{1/2} \tag{5}$$

According to the model definitions (Alonso et al. 1990), the p_0 and p_s are given as

$$p_0 = p^c(p_0^*/p^c)^{[\lambda(0)-\kappa]/[\lambda(s)-\kappa]} \tag{6}$$

$$p_s = ks \tag{7}$$

where $\lambda(s) = \lambda(0)[(1 - r)\exp(-\beta s) + r]$. Note that p_0^* is a hardening parameter that depends on the plastic volumetric strain ε_v^p and can be expressed as

$$\Delta p_0^* = p_0^* \frac{1 + e_0}{\lambda(0) - \kappa} \Delta\varepsilon_v^p \tag{8}$$

where e_0 = initial void ratio. The plastic potential function in (3), Q, is taken as equal to the yield function in this paper and then associated plasticity can be considered. The vectors of $\mathbf{h}_e$ and $\mathbf{h}_p$ in (3) can be expressed as

$$\mathbf{h}_e = \frac{\kappa_s}{3(1 + e_0)(s + u_{atm})} \mathbf{I} \tag{9}$$

$$\mathbf{h}_p = \frac{(\lambda_s - \kappa_s)}{3(1 + e_0)(s + u_{atm})} \mathbf{I} \quad \text{if } F_0 = s - s_0 > 0 \tag{10a}$$

$$\mathbf{h}_p = 0 \quad \text{if } F_0 = s - s_0 \leqq 0 \tag{10b}$$

where u_{atm} = standard atmospheric pressure (101.4 kPa); s_0 is second hardening parameter related to second yield surface, i.e., $F_0 = 0$; and λ_s and κ_s together with the aforementioned $\lambda(0)$, r, β, k, κ, p^c, p_0^*, and M = model parameters that can be determined by laboratory tests (Alonso et al. 1990).

In some cases, the thermal effect on deformation behavior of saturated and unsaturated soils can be significant. Experimental works indicate that at varying stress levels, soils may either expand or contract upon heating (Campanella and Mitchell 1968; Demars and Charles 1982; Agar et al. 1986; Hueckel and Pellegrini 1989; Hueckel and Baldi 1990; Romero et al. 1995). This phenomenon is related to thermal softening and thermal plasticity. Based on the test results, Hueckel and Pellegrini (1989) and Hueckel and Baldi (1990) have pioneered the development of a thermoplastic constitutive model for saturated soils. However, information on the thermal behavior of unsaturated soil is very limited. Until more data become available, the validation of a comprehensive constitutive model including unsaturated and thermal effects will be difficult. For this reason, this paper only accounts for linear thermal volumetric strains by using a constant linear thermal expansion coefficient β^T.

Governing Equations for Pore Moisture

The pore-moisture is composed of the pore-liquid water and the pore-water vapor. The mass flux of nonisothermal pore-liquid water can be defined by a generalized form of Darcy's law (Bear and Verruijt 1987) as

$$\mathbf{q}_w = -\frac{\mathbf{k}_w}{g} \nabla(u_w + \rho_w g z) \tag{11}$$

where $\mathbf{q}_w$ = mass flux vector of the pore-liquid water; $\mathbf{k}_w$ = unsaturated permeability matrix of the liquid water phase; g = acceleration due to gravity (9.81 m/s^2); z = vertical space coordinate (positive is upward); and ρ_w = density of water. The unsaturated permeability of the liquid water phase is dependent upon the porosity, the liquid water content or the suction of the soil, and also upon the temperature for nonisothermal problems. The unsaturated permeability can be formulated as

$$\mathbf{k}_w(t^0, s, n) = \frac{\mu_w(T_r)}{\mu_w(t^0 + 273.16)} \mathbf{k}_{ws}^r(n) k_{rw}^r(s) \tag{12}$$

where n = porosity of the soil; μ_w = dynamic viscosity of water [i.e., $0.6612(44.16 + t^0)^{-1.56}$ (Nsm^{-2})] (Thomas and Li 1995); T_r = absolute room temperature (K); t^0 = temperature (°C); $\mathbf{k}_{ws}^r$ = saturated permeability matrix related to the porosity of the soil at room temperature; and k_{rw}^r = relative unsaturated permeability associated with the suction of the soil at room temperature.

In (11), the pore-water pressure can be rewritten as $u_w = -(u_a - u_w) + u_{ad} + u_v$. Substituting the pore-water pressure into (11) yields

$$\mathbf{q}_w = -\frac{\mathbf{k}_w}{g} \nabla[-(u_a - u_w) + u_{ad} + u_v + \rho_w g z] \tag{13}$$

where the matric suction, s [i.e., $(u_a - u_w)$] is, under nonisothermal conditions, dependent upon the temperature and the volumetric liquid water content, θ_w, or the degree of saturation, S. The effect of the temperature on the matric suction can be calculated by using surface tension models as

$$s(t^0 + 273.16, \theta_w) = \frac{E(t^0 + 273.16)}{E(T_r)} s_r(\theta_w) \tag{14}$$

where $E(t^0 + 273.16)$ = surface energy of liquid water, i.e., $0.1171 - 1.516 \times 10^{-4}(t^0 + 273.16)$ (Jm^{-2}) (Thomas and Li 1995).

Eq. (13) clearly shows that the mass flux of the pore-liquid water is not only dependent upon the matric suction gradient, but also dependent upon the pore-dry air pressure gradient and the pore-vapor pressure gradient. Therefore, the matric suction-head based formulation is not applicable to analysis of heat and moisture transfer in saturated and unsaturated soils. The mass flux of the pore vapor can be considered to occur due to two effects, first, under molecular diffusion and, second, as part of the bulk flow of the pore-air. The mass flux of the pore-vapor due to molecular diffusion can be described using Fick's Law (Philip and de Vries 1957; de Vries 1975; Fredlund and Dakshanamurthy 1982). Therefore the total mass flux of the pore-vapor can be expressed as

$$\mathbf{q}_v = -D_{atm}\tilde{\alpha}\tilde{\beta}\nabla\rho_v + \frac{\rho_v}{\rho_a}\mathbf{q}_a \tag{15}$$

where D_{atm} = molecular diffusivity of the pore-vapor in air [i.e., $2.29 \times 10^{-5}(1 + t^0/273.16)^{1.75}$ (m^2/s)] (Kimball and Jackson 1976); $\tilde{\alpha}$ = tortuosity factor for soil (i.e., $\tilde{\beta}^{2/3}$) (Lai et al. 1976); $\tilde{\beta}$ = cross-sectional area of the soil that is available for vapor flow [i.e., $(1 - S)n$]; $\mathbf{q}_a$ = mass flux of the pore-air due to the air pressure gradient; and ρ_v and ρ_a = densities of the pore-vapor (i.e., $\rho_v^0 h_r$) and the pore-air (i.e., $\rho_{ad} + \rho_v$), respectively. Assuming that the pore-liquid water is in equilibrium with the pore-water vapor (de Vries 1958), and both the pore-dry air and the pore-water vapor are ideal gases (Pollock 1986), the following density equations can be written as

$$\rho_v = \frac{M_w u_v}{R(t^0 + 273.16)} \quad (16)$$

$$\rho_{ad} = \frac{M_{ad}(u_{atm} + u_{ad})}{R(t^0 + 273.16)} \quad (17)$$

$$\rho_a = \frac{M_a(u_{atm} + u_a)}{R(t^0 + 273.16)} \quad (18)$$

where R = universal gas constant [8.314 J/(mol·K)]; M_{ad} = molecular weight of the dry air (0.02895 kg/mol); M_w = molecular weight of water (0.018 kg/mol); M_a = molecular weight of air, i.e., $[M_{ad}(u_{atm} + u_{ad}) + M_w u_v]/(u_{atm} + u_{ad} + u_v)$ (kg/mol); ρ_v^0 = density of saturation water vapor, i.e., $M_w u_v^0/R(t^0 + 273.16)$ (kg/m^3); u_v^0 = partial pressure of saturation water vapor (i.e., $1.36075 \times 10^5 \exp[-5{,}239.7/(t^0 + 273.16)]$ (MPa) (Olivella 1995); h_r = relative humidity, i.e., $\exp[-(u_a - u_w)M_w/\rho_w R(t^0 + 273.16)]$; and u_v = partial pressure of the pore vapor, i.e., $u_v^0 h_r$. The mass flux of the pore-air due to the air pressure gradient $\mathbf{q}_a$ can be computed by applying the generalized form of Darcy's law for multiphase flow (Barden 1965); therefore, ignoring gravitational effects, the mass flux is given as

$$\mathbf{q}_a = -\mathbf{k}_a \frac{\rho_a \nabla u_a}{\rho_w g} \quad (19)$$

where $\mathbf{k}_a$ = unsaturated permeability matrix of the air phase that is defined as

$$\mathbf{k}_a = \frac{\mu_a(T_r)}{\mu_a(t^0 + 273.16)} \mathbf{k}_{as}^r(n) k_{ra}^r(s) \quad (20)$$

where μ_a = dynamic viscosity of the pore-air, i.e., $1.48 \times 10^{-6}(t^0 + 273.16)/[1 + 119/(t^0 + 273.16)]$ (Ns m^{-2}) (Rossel and Jeanet 1970); $\mathbf{k}_{as}^r$ = permeability matrix for the air phase related to the porosity of the soil at dry and room temperature; and k_{ra}^r = relative unsaturated permeability associated with the matric suction of the soil at room temperature. Substituting (16) and (19) into (15) gives

$$\mathbf{q}_v = D_{atm}\tilde{\alpha}\tilde{\beta} \frac{M_w \rho_v}{R(t^0 + 273.16)\rho_w} \nabla(u_a - u_w) - D_{atm}\tilde{\alpha}\tilde{\beta}\tilde{\xi}\left(h_r \frac{d\rho_v^0}{dT} + \frac{M_w(u_a - u_w)\rho_v}{R(t^0 + 273.16)^2\rho_w}\right)\nabla T - \mathbf{k}_a \frac{\rho_v}{\rho_w g}\nabla u_a \quad (21)$$

where $\tilde{\xi}$ = ratio of microscopic temperature gradient in the pore space to the macroscopic temperature gradient, i.e., $(\nabla T)_a/\nabla T$ and can be assumed to be 1.0. Applying the principle of mass conservation to the moisture phase yields

$$\frac{\partial}{\partial t}[\rho_w nS + \rho_v n(1 - S)] = -\nabla\cdot(\mathbf{q}_w + \mathbf{q}_v) + f^w \quad (22)$$

where $n = n_0 - \varepsilon_v = n_0 + (\partial u/\partial x) + (\partial v/\partial y) + (\partial w/\partial z)$; u, v, and w = three components of displacement in the x-, y-, and z-directions, respectively; ε_v and n_0 = volumetric strain and the initial porosity, respectively; and f^w = evapotranspiration mass rate per unit of soil volume due to the uptake of water by the evaporation process and the roots of plants or external mass supply rate of the moisture per unit volume of soil.

Governing Equations for Pore-Dry Air

The pore-dry air is composed of the dry air within the pore air and the dry air dissolved in the pore liquid water. The transfer of the pore-dry air is assumed to take place due to two effects: first, as part of the bulk flow of the pore-air under the air pressure gradient, and second, within the pore-liquid water under the influence of the pore-water pressure gradient. The volumetric mass of dissolved dry air contained within the liquid water can be obtained by the use of Henry's volumetric coefficient of solubility (Weast 1976). The application of the law of conservation of mass for the pore-dry air phase therefore yields

$$\frac{\partial}{\partial t}[(1 - S)n\rho_{ad} + \bar{H}Sn\rho_{ad}^*] = -\nabla\cdot(\mathbf{q}_a^{ad} + \mathbf{q}_w^{ad}) + f^{ad} \quad (23)$$

where f^{ad} = external mass supply rate of the dry air per unit volume of soil; the density of the dissolved dry air, ρ_{ad}^*, can be assumed to be the density of the pore-dry air (i.e., ρ_{ad}) for simplicity; and the Henry's volumetric coefficient, $\bar{H}$, is equal to 0.02 under standard atmospheric pressure (101.4 kPa) and room temperature (20°C). The dry air mass flux in the pore-air phase, $\mathbf{q}_a^{ad}$, and the dry air mass flux in the pore-liquid water phase, $\mathbf{q}_w^{ad}$, can be given as

$$\mathbf{q}_a^{ad} = -\mathbf{k}_a \frac{\rho_{ad}}{\rho_w g}\nabla u_a \quad (24)$$

$$\mathbf{q}_w^{ad} = -\bar{H}\mathbf{k}_w \frac{\rho_{ad}^*}{\rho_w g}\nabla(u_w + \rho_w g z) \quad (25)$$

Governing Equations for Heat

In general, the heat transfer in unsaturated soils is governed by the thermal conduction in accordance with Fourier's law. The factors that affect the heat transfer are the convection of sensible heat in the pore-moisture phase and in the pore-dry air phase, and the latent heat flow in the pore-water vapor. The energy flux of heat can therefore be given as

$$\mathbf{Q} = -\lambda\nabla T + C_{pw}(T - T_0)\mathbf{q}_w + C_{pv}(T - T_0)\mathbf{q}_v + C_{p,ad}(T - T_0)(\mathbf{q}_a^{ad} + \mathbf{q}_w^{ad}) + L_v\mathbf{q}_v \quad (26)$$

where C_{pw} = specific heat capacity of the pore liquid water, i.e., 4,180 (Jkg^{-1}K^{-1}) (Mayhew and Rogers 1976); C_{pv} = specific heat capacity of the pore-vapor, i.e., 1,870 (Jkg^{-1}K^{-1}) (Mayhew and Rogers 1976); $C_{p,ad}$ = specific heat capacity of the pore "dry" air, i.e., 1,000 (Jkg^{-1}K^{-1}) (Olivella 1995); L_v = latent heat of vaporization of the liquid water, i.e., 2.418×10^6 (Jkg^{-1}) (Mayhew and Rogers 1976); T_0 = absolute reference temperature (K); and C_{ps} = specific heat capacity of the soil solids and is dependent on the types and characteristics of the soil solids.

The thermal conductivity of an unsaturated soil, λ, can be determined using the method described by de Vries (1963) (Fredlund and Rahardjo 1993)

$$\lambda = \frac{f_s\theta_s\lambda_s + f_w\theta_w\lambda_w + f_a\theta_a\lambda_a}{f_s\theta_s + f_w\theta_w + f_a\theta_a} \quad (27)$$

where f_s, f_w, and f_a = weighting factors for the soil solids, the pore-liquid water, and the pore-air, respectively; λ_s, λ_w, and λ_a = thermal conductivities of the soil solids, the pore-liquid water (i.e., 0.57 Wm^{-1}K^{-1}), and the pore-air, respectively; $\lambda_a = \lambda_{ad} + \lambda_v$; λ_{ad} = thermal conductivity of the pore-dry air, i.e., 0.0258 (Wm^{-1}K^{-1}); λ_v = thermal conductivity of the pore-vapor, i.e., $D_{atm}h_r L_v(d\rho_v^0/dT)$ (Wm^{-1}K^{-1}); λ_s is mainly dependent on the type and properties of the soil solids; and θ_s, θ_w, and θ_a = volumetric fractions of the soil solids, the pore-liquid water, and the pore-air, respectively. The determination of f_s, f_w, and f_a and calculation of λ can be found in de Vries (1963), Jame (1977), and Fredlund and Rahardjo (1993). Applying the law of energy conservation yields

$$\frac{\partial}{\partial t}\Phi = -\nabla\cdot\mathbf{Q} + f^{\Phi} \quad (28)$$

where f^{Φ} = internal and/or external energy supply rate per unit volume of soil; and the total internal energy of heat in the soil, Φ, can be expressed in the following equation by assuming that the differential heat of wetting of the soil is negligible:

$$\Phi = C_{ps}\rho_s(1-n)(T-T_0) + C_{pw}\rho_w Sn(T-T_0) + C_{p,ad}[\rho_{ad}(1-S)n + \bar{H}\rho^*_{ad}Sn](T-T_0) + C_{pv}\rho_v(1-S)n(T-T_0) + L_v\rho_v(1-S)n \quad (29)$$

The total internal energy of heat in the soil, Φ, consists of two terms. One represents the storage of latent heat due to the accumulation of the pore-vapor, and the other represents the local capacity for heat comprising the capacity of the soil solids, the pore-liquid water, the pore-dry air, and the pore-water vapor.

A general mathematical model for solving coupled heat, moisture, dry air flow, and deformation problems in unsaturated soils can be formulated by combining (1), (22), (23), and (28) and applying necessary initial and boundary conditions (i.e., either Dirichlet or Neumann type). This model contains six independent variables as the essential unknown variables: displacement, u, v, and w; pore-water pressure, u_w; pore-air pressure, u_a; and absolute temperature, T (i.e., t^0 + 273.16).

Interaction between Different Phases

In an unsaturated soil, there are three basic phases: solid, liquid water, and air phases. The air phase is composed of two continuous subphases, vapor phase and dry air phase. In general, the interaction between the different phases is complex. In a nonisothermal condition, the pore-liquid water transfer is under a water pressure gradient and the pore-dry air transfer is under an air pressure gradient. The vapor transfer is due to two effects: first, the molecular diffusion, and second, as part of the bulk flow of the pore-air. Both effects are subjected to changes in matric suction, temperature, air pressure, and deformation of soil structure as well as flux boundaries. Meanwhile, the pore-liquid water flow, the pore-vapor flow, and the pore-dry air can cause redistribution of water content, temperature, and thermal energy; and dissipation and redistribution of pore-water pressure, pore-vapor pressure, and pore-dry air pressure. The dissipation and redistribution in return can induce the changes in matric suction, deformation, stress, stiffness, and shear strength of the soil and changes in relative humidity, vapor density, and vapor pressure, as well as changes in temperature, thermal energy, and other soil properties.

The heat transfer as governed by thermal conduction, convection of sensible heat in pore-liquid water phase, in pore-vapor phase, and in pore-dry air phase, and latent heat flow in the pore-vapor phase is related to the changes in matric suction, temperature, air pressure, and deformation of soil structure as well as flux boundaries. On the other hand, the high temperature gradient can cause both liquid and vapor to be driven away from the source of high temperature, and the liquid transfer provides a longer component of moisture flow from the source of high temperature than vapor transfer component. The temperature and thermal energy changes can lead to either softening or hardening, as well as thermoplastic yield in unsaturated soils. The temperature and thermal energy changes can also affect matric suction, pore-vapor pressure, pore-liquid pressure, pore-dry air pressure, and other soil properties.

The deformation of soil structure upon changes in loading and/or matric suction change and/or temperature change may induce further changes in matric suction, pore-vapor pressure, pore-liquid pressure, pore-dry air pressure, and other soil properties.

Material Parameters

For the convenience of discussion, (3) can be rewritten in a simple form as

$$\Delta\boldsymbol{\sigma}^* = \mathbf{D}_{ep}\Delta\boldsymbol{\varepsilon} - \mathbf{h}_{ep}\Delta s + \mathbf{D}_{ep}\beta^T\mathbf{I}\Delta T \quad (30)$$

where $\mathbf{D}_{ep}$ = elastoplastic matrix; and $\mathbf{h}_{ep}$ = elastoplastic matric suction coefficient vector. The parameters involved in (30) have already been discussed above. If the nonlinear elastic constitutive model of unsaturated soils is used, the matrix $\mathbf{D}_{ep}$ and the vector $\mathbf{h}_{ep}$ in (30) can be reduced to a nonlinear elastic matrix $\mathbf{D}_{ne}$ and a nonlinear matric suction coefficient vector $\mathbf{h}_{ne}$, respectively. To date, most of the nonlinear elastic constitutive models (Lloret and Ledesma 1993; Fredlund and Rahardjo 1993) consider only volumetric strains due to matric suction changes, but do not consider distortional strains due to matric suction changes. Yang and Shen (1992) proposed a generalized nonlinear constitutive model that can simulate four essential characteristics of unsaturated soils, such as nonlinearity, volumetric and distortional strains due to matric suction changes, dilatancy, and creep. Based on the model proposed by Yang and Shen (1992) and assuming that the creep effect is negligible here, (30) can be modified to include the thermal effect and can be rewritten as

$$\Delta\boldsymbol{\sigma}^* = \mathbf{D}_{ne}\Delta\boldsymbol{\varepsilon} - (d_4\mathbf{I} + d_5\mathbf{J})\Delta s + d_6\beta^T\mathbf{I}\Delta T + d_7\mathbf{I}\Delta q \quad (31)$$

where q = generalized shear stress, i.e., $\sqrt{6}\tau_\sigma/2$; τ_σ = π-plane shear stress given in (5); $\mathbf{J} = [\sigma_x - \sigma_m, \sigma_y - \sigma_m, \sigma_z - \sigma_m, \tau_{xy}, \tau_{yz}, \tau_{zx}]^T/\tau_\sigma$; and $\mathbf{D}_{ne}$ = nonlinear elastic matrix and can be expressed as

$$\mathbf{D}_{ne} = [d_{ij}] = \begin{bmatrix} d_1 & d_2 & d_2 & 0 & 0 & 0 \\ d_2 & d_1 & d_2 & 0 & 0 & 0 \\ d_2 & d_2 & d_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & d_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & d_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & d_3 \end{bmatrix} \quad (i, j = 1 \sim 6) \quad (32)$$

where $d_1 = K_t + 4G_t/3$; $d_2 = K_t - 2G_t/3$; $d_3 = G_t$; $d_4 = K_t\rho^s_t$; $d_5 = G_t\rho^\gamma_{s,t}$; $d_6 = 3K_t$; $d_7 = -K_t\rho^d_t$; and ρ^s_t, $\rho^\gamma_{s,t}$, and ρ^d_t = volumetric collapse coefficient, distortional collapse coefficient, and volumetric dilatant coefficient, respectively. The determination of the five parameters K_t, G_t, ρ^s_t, $\rho^\gamma_{s,t}$, and ρ^d_t was discussed in detail by Yang and Shen (1992). The tangent bulk modulus of the soil, K_t, involved in the matrix $\mathbf{D}_{ne}$ can be obtained from the void ratio state surface constitutive equations under isotropic test conditions as

$$\frac{1}{K_t} = \frac{1}{1+e}\frac{\partial e(\sigma - u_a, u_a - u_w)}{\partial(\sigma - u_a)} \quad (33)$$

Lloret and Alonso (1985) analyzed eight possible analytical expressions for the state surface by using data-fitting techniques and found that the following expression is suitable for use:

$$e = a_e + b_e\log(\sigma - u_a) + c_e\log(u_a - u_w + u_{atm}) + d_e\log(\sigma - u_a)\log(u_a - u_w + u_{atm}) \quad (34)$$

where e = void ratio; σ = relevant total stress; and a_e, b_e, c_e, and d_e = constants. The shear modulus of an unsaturated soil G_t can be obtained from a hyperbolic shear stress-strain law under triaxial test conditions as

$$G_t = [G_0 + M_s(u_a - u_w)]\left[1 - \frac{(\sigma_1 - \sigma_3)R_f}{(\sigma_1 - \sigma_3)_f}\right]^2 \quad (35)$$

where G_0 (i.e., initial shear modulus) and R_f (i.e., failure ratio) = parameters of the hyperbolic model (Duncan and Chang 1970). The stiffening effect due to an increase in matric suction has been introduced in (35) through the addition of a parameter, M_s, to the initial shear modulus, G_0 (Lloret and Ledesma 1993). The following expression proposed by Fredlund et al. (1978) can be used as a failure criterion:

$$\tau_{ff} = c' + (\sigma_n - u_a)\tan\phi' + (u_a - u_w)\tan\phi^b \quad (36)$$

where τ_{ff} = shear stress at failure; c' = effective cohesion; ϕ' = effective internal friction angle associated with the net normal stress state variable; ϕ^b = angle indicating the rate of change in shear strength relative to the change in matric suction; and $(\sigma_n - u_a)$ = net normal stress on the failure surface.

From the void ratio state surface constitutive equation, the volumetric collapse coefficient can be derived as

$$\rho_t^s = \frac{1}{1+e}\frac{\partial e(\sigma - u_a, u_a - u_w)}{\partial(u_a - u_w)} \quad (37)$$

If the dilatant effect and distortional collapse effect can be assumed to be negligible for simplicity here, the coefficients of d_5 and d_7 in (31) are then equal to zero, and the volumetric collapse coefficient can also be simply expressed as

$$\rho_t^s = \partial\varepsilon_v^w/\partial(u_a - u_w) \quad (38)$$

where ε_v^w = volumetric strain due to suction changes that may be equal to the volumetric strain due to suction changes under isotropic triaxial compression test condition with constant net confining pressure or equal to the vertical strain due to suction changes under oedometer test condition with constant load. The thermal volumetric strain can be simulated through a constant linear thermal expansion coefficient, β^T. The last term in (31) is the dilatant effect of changing of the generalized shear stress. The parameter relative to the dilatant effect, ρ_t^d, can be determined from triaxial compression test. As for the distortional collapse coefficient relative to the distortional strain due to matric suction changes, $\rho_{s,t}^\gamma$, can be obtained from triaxial compression test with constant suction condition.

The saturated permeability matrix for the pore-liquid water phase at room temperature, $\mathbf{k}_{ws}^r(n)$, the unsaturated relative permeability for the pore-liquid water phase at room temperature, $k_{rw}^r(s)$ or $k_{rw}^r(S)$, the permeability matrix for the pore-air phase under dry and room temperature conditions, $\mathbf{k}_{as}^r(n)$, and the unsaturated relative permeability for the pore-air phase, $k_{ra}^r(s)$ or $k_{ra}^r(S)$, can be obtained from laboratory tests. The $\mathbf{k}_{ws}^r(n)$ can be assumed to be constant in many cases. Many empirical expressions of $k_{rw}^r(s)$ or $k_{rw}^r(S)$ are available (Alonso et al. 1987). To date, only a few empirical expressions for $\mathbf{k}_{as}^r(n)$ and $k_{ra}^r(s)$ or $k_{ra}^r(S)$ have been proposed (Yoshimi and Osterberg 1963; Brun 1989; Alonso et al. 1988). Based on the definition in (19) and the empirical expressions proposed by Alonso et al. (1988), $\mathbf{k}_a$ in (20) can be redefined as

$$\mathbf{k}_a = \frac{\mu_a(T_r)}{\mu_a(t^0 + 273.16)}\mathbf{k}_a^r(e, S) \quad (39)$$

$$\mathbf{k}_a^r(e, S) = \frac{\mathbf{B}_a\rho_w g}{\mu_a(T_r)}[e(1 - S)]^{\beta_a} \quad (40)$$

where β_a = constant; and $\mathbf{B}_a$ = constant matrix. If the soil is isotropic, $\mathbf{B}_a$ can be reduced to a constant. The values of $\mathbf{B}_a$ and β_a can be determined from curve-fitting techniques applied to the experimentally determined data. If there is no available experimental data, $\mathbf{k}_a$ can be defined theoretically as

$$\mathbf{k}_a = \frac{\mu_a(T_r)}{\mu_a(t^0 + 273.16)}\mathbf{k}_a^r(n, s) \quad (41)$$

$$\mathbf{k}_a^r(n, s) = \frac{\mu_w(T_r)}{\mu_a(T_r)}\mathbf{k}_{ws}^r(n)k_{ra}^r(s) \quad (42)$$

Besides stress-strain behavior and the permeabilities for the pore-liquid water phase and the pore-air phase, the degree of saturation is another important soil parameter. The degree of saturation, S, is related to matric suction and stress state. The state surface with respect to the degree of saturation can be determined from experimental tests. Lloret and Alonso (1985) proposed several empirical expressions for the state surface with respect to the degree of saturation that can be obtained using data-fitting techniques. The following general relationship between volumetric water content and matric suction was proposed by Fredlund and Xing (1994):

$$\theta_w = C(u_a - u_w)\frac{\theta_{ws}}{\{\ln[e + [(u_a - u_w)/a]^n]\}^m} \quad (43)$$

where $C(u_a - u_w)$ = correction function defined as

$$C(u_a - u_w) = 1 - \frac{\ln[1 + (u_a - u_w)/(u_a - u_w)_r]}{\ln[1 + 1{,}000{,}000/(u_a - u_w)_r]}$$

where $(u_a - u_w)_r$ (kPa) = matric suction corresponding to the residual volumetric water content θ_r; a, n, and m = three material parameters; and θ_{ws} = saturated volumetric water content.

As discussed previously, the properties of phases appeared in the balance equations, i.e., density, viscosity, surface tension, molecular diffusivity, enthalpy, etc., are in general considered dependent on the composition of the phase and on the independent variables (temperature, pressure, etc.). The foregoing empirical or rational equations for density, viscosity, surface tension, molecular diffusivity, and enthalpy are considered to be adoptable in the proposed coupled model in this paper.

NUMERICAL METHOD

In this paper, the Galerkin finite-element method is used to discretize (1), (22), (23), and (28), and the formulation

$$\int_{t_k}^{t_k+\Delta t}(\ )\,dt \Rightarrow \Delta t[(1 - \alpha)(\ )_{t_k} + \alpha(\ )_{t_k+\Delta t}]$$

is applied to discretize the time domain, where Δt = time step; $(\ )_{t_k}$ and $(\ )_{t_k+\Delta t}$ = values of the specified function at time t_k and $t_k + \Delta t$, respectively; and α = integral parameter. If the increments of the displacements Δu, Δv, Δw, the pore water pressure Δu_w, the pore air pressure Δu_a, and the temperature ΔT are accepted as the independent variables, the finite-element formulas for solving coupled heat, moisture, air flow, and deformation problems in unsaturated soils can be expressed as

$$\sum_{j=1}^{N}[\mathbf{k}_{ij}]^{t_k+0.5\Delta t}\{\Delta V_j\} = \{\Delta F_i\} \quad (i = 1, 2, 3, \ldots, N) \quad (44)$$

where $\{\Delta V_j\} = [\Delta u_j, \Delta v_j, \Delta w_j, \Delta u_{wj}, \Delta u_{aj}, \Delta T_j]^T$ = vector of increments of the node variables at node j within time interval Δt; $\{\Delta F_i\} = [\Delta f_i^1, \Delta f_i^2, \Delta f_i^3, \Delta f_i^4, \Delta f_i^5, \Delta f_i^6]^T$ = vector of increments of the generalized equivalent nodal forces, moisture mass, air mass, and heat energy at node i within time interval Δt; the elements of the submatrix $\mathbf{k}_{ij}$, k_{ij}^{11}, k_{ij}^{12}, ..., k_{ij}^{66}, can be derived using the finite-element technique; N = total number of nodes; and $(\ )^{t_k+0.5\Delta t}$ = values of the specified function at time t_k + (1/2)Δt. The detailed derivation of the vector of increments of the generalized equivalent nodal forces, moisture mass, air mass, and heat energy at node i within time interval Δt, $\{\Delta F_i\}$, as well as the submatrix $\mathbf{k}_{ij}$ in (44), is not difficult but very lengthy and tedious. Therefore, due to limitations of space, the tedious details for deriving the vector $\{\Delta F_i\}$ and the submatrix $\mathbf{k}_{ij}$ in (44) as well as the composed final generalized stiffness matrix are not presented here.

Based on the formulas described in this paper, a 3D finite-element computer program, C-HWAM-3D, for analyses of coupled heat, moisture, and air transfer in deformable unsaturated soils is developed. Two kinds of spatial isoparametric elements, i.e., eight-noded brick and six-noded prism isoparametric elements, are incorporated in this program. Since the resulting set of global matrices is nonsymmetric, the solution of the corresponding resulting set of nonlinear algebraic equations is therefore achieved by using a so-called nonsymmetric direct solution algorithm within this new program. An iterative technique is used to solve the obtained nonlinear algebraical equations and the convergence of solution is assumed to achieve when the difference between successive iterations falls within a specified tolerance. As the discussion on the so-called nonsymmetric direct solution algorithm and the iterative technique is beyond the scope of this paper and also very lengthy and tedious, the details are not presented here.

The calculation of deformation characteristics of unsaturated soils [i.e., the constitutive (3)] can account for not only the effect of matric suction and stress level, but also the effect of temperature. The deformation due to matric suction changes in unsaturated soils is a very important and also difficult topic. The change in matric suction of an unsaturated soil at constant temperature is mainly due to two factors: the compression or expansion of the soil structure and the change in water content. Test results (Alonso et al. 1995) have shown that there is an essential difference between the matric suction changes due to these two different factors. For a given unsaturated expansive clay with the same initial conditions, only the matric suction change due to a change in water content of the soil will result in swelling deformation, but the matric suction change due to loading should not lead to swelling deformation. This example indicates clearly that it is necessary and important to differentiate the matric suction effects on deformation. For application to practical problems, an equation needs to be established to distinguish the matric suction changes induced by these two different factors as follows:

$$\text{If } |\theta_w - \theta_{wo}| < \varepsilon_\theta, \text{ then } (x, y, z) \in \Omega_{UW};$$

$$\text{If } |\theta_w - \theta_{wo}| \geq \varepsilon_\theta, \text{ then } (x, y, z) \in \Omega_W \quad (45)$$

where ε_θ = error tolerance; θ_{wo} = initial volumetric water content; and Ω_{UW} and Ω_W = nonwetted zone and wetted zone, respectively. Within the nonwetted zone, Ω_{UW}, the effect of matric suction changes on deformation is assumed to be negligible.

CONCLUSIONS

Through extensive review and study of the related achievements made in different aspects of coupled processes in unsaturated soil, a general 3D mathematical model for fully coupled heat, moisture, air flows, and deformation problems in unsaturated soils has been proposed to describe the different aspects of the unsaturated soil behavior in a consistent and unified manner. In the model, the pore-water and air transfers were assumed to be governed by Darcy's law and the pore-vapor transfer was considered to occur due to two effects: first, under the molecular diffusion, and second, as part of the bulk flow of the pore air. The pore-vapor transfer due to the molecular diffusion was described using Fick's law. The heat transfer was formulated to include the effects of conduction, convection, and latent heat of vaporization. An elastoplastic constitutive framework was used to describe the deformation behavior of the unsaturated soil skeleton where the linear elastic and nonlinear elastic models are just two special cases. In particular, a generalized nonlinear constitutive model proposed earlier by the first writer was introduced and incorporated into the framework to predict the soil deformation with modification to account for the effect of temperature changes on deformation. The determination of the soil parameters involved in the coupled model was carefully discussed. A set of fully coupled, nonlinear partial differential equations were established and then solved by using a Galerkin weighted residual approach in the space domain and using an implicit integrating scheme in time domain. A robust new 3D computer program, C-HWAM-3D, has been developed to incorporate the proposed model for analyzing the transient coupled flows of heat, moisture, and air, and the stress and strain in unsaturated soils.

In order to check the reliability and capability of the coupled model as well as the new computer code developed in this paper, validations should therefore be made accordingly through comparisons between the theoretical numerical solutions and the corresponding test measurements or analytical results. However, due to limitations of space in this paper, the detailed validations for the theoretical work that have been carried out will be presented in another paper. Good agreement between the computed numerical results and the measurements of several cases from published literature confirms the reliability and capability of the model. Results of the validations indicate that the model is general and suitable for the analyses of many different problems, such as isothermal or nonisothermal multiphase flows in deformable or nondeformable saturated-unsaturated soils, seepage in saturated-unsaturated soils, and deformation characteristics with consideration of elastoplastic or nonlinear elastic behavior in saturated-unsaturated soils.

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necessary, and they would be hampered by the 120 days a year operating limit. To save time, the solution had to be sought in optimizing existing designs and in the use of as many standard components as possible. After careful evaluation, a newly launched Dutch plain suction dredger, the "IJsselmeer", met the conditions of this project. The "IJsselmeer" dredger works excellently in the soil where only cutter dredger could work previously. In addition, the soil at Mailiao is even easier for the "IJsselmeer" dredger to dredge than the soils that the "IJsselmeer" dredger used to work in.

The major design concept of the plain suction dredgers follows those of the "IJsselmeer" dredger. The major dimensions and power plant of the dredger are shown in Table 2. An underwater pump is connected at the bottom of the ladder. The pump is driven by deck-mounted engines that operating through a pivoting gearbox. The pivoting gearbox is of heavy duty and moves with the ladder. The engines are easy to maintain because of mounted on the deck rather than mounted below the deck. Other merits of this type of dredger are high fuel efficiency, ease of control, longevity and relatively low cost. Booster pumps were deemed necessary to bridge the large pumping distance. Two pump drives were chosen of the type used in the Beaver 3800 cutter dredgers. The double walled pumps had to be modified to withstand an end pressure of 24 bar.

Table 2 Major dimensions and power plant of the dredger

Major dimension:	
Overall length	72.1 m
Beam	14.9 m
Depth at side	4.25 m
Average draught (approx.)	2.25 m
Max. dredging depth (approx.)	34 m
Diameter of the suction tube	700 mm
Diameter of discharge pipe	700 mm
Power plant:	
For the submerged dredge pump	1,752 kW
For the dredge pumps on deck	(2x) 1,908 kW
For the jet pumps	876 kW
For on board network generator and hydraulic pumps	876 kW
For the harbor set	105 kW
Total installed power	7,425 kW

Optimization of Design

In order to increase production and bring down the cubic meter price, the average annual operational days had to be doubled. The target became 240 or even 250 days. In other words, the dredgers had to be able to dredge under heavy sea weather. Experience with seaworthy stationary dredgers was limited. Statistics for the area indicated that 2.75m could be assumed as a significant wave height, with a period of 7 to 9 seconds, which happens to be a dangerous long swell. If a craft could be designed to meet these conditions, only two dredgers would be needed for finishing the 74-million-cubic-meter fills in 4.5 years. Both the contractor and the dredger builder agreed to design the dredgers base on the conditions described previously with the considerations of winning sand from both shallow and deep water, optimizing the size of the craft, coupling the floating pipelines and providing ways for swell compensation for the suction pipe.

The design considerations of the suction pipe and the swell compensation system are as following:

1) The dredging operation was to begin in very shallow water.
2) Optimum production required to be able to dredging as deep as the authorities would allow. (Production of a plain suction dredger increases with the depth of the pit).
3) The system must be able to follow vertical and horizontal movements of about 10 m and yet retain enough stability to avoid production losses.
4) Compensation should preferably not be sought in the heavy wheel ladder, which would be complicated and hence prohibitively expensive.
5) The suction tube must be able to dredge through layers of consolidated silt.
6) The system should be able to survive the collapsing of deep pit.
7) The system must be able to free itself by continuing the dredging process as well as by pulling the suction tube free in a controlled manner and should not need emergency hoisting gear.
8) The system should be able to reduce the mixture concentration when pumping over long distance.
9) Servicing the suction head in heavy seas should be possible.
10) The system should be simple to operate.

Floating Pipeline

In order to fulfill above-mentioned requirements, special design features of the floating pipeline connection and the anchoring system were incorporated in the dredger design. Coupling the floating pipelines to a large extent dictates the maximum wave height in which the dredger can work. This could be done near the center of floatation. An additional advantage was that the pipeline would be well out of the way of the anchor cables and reduced the risk of damage significantly. The craft is designed to meet the Northeast wind on the stern, and the floating pipeline coupling meets the wind under a certain angle. The funnel-shaped support, built of anti-friction materials, made the coupling procedure easier and prevented kinks. The coupling can be used on the port as well as on the starboard sides. For working in heavy sea conditions, a quick coupling device and a heavy winch were incorporated. The heaviest anchor winch is on the stern. Calculations indicated that an anchor cable of at least 700m long would need a winch of only 480 kN. Furthermore, the craft has 4 side winches on the quarters and also a bow winch. The floating pipeline had a anchoring points every 70 meter.

Swell Compensation

The passive swell compensation system is another particular feature of the plain suction dredgers. Because the active swell compensation must be fed from sensors and needs a lot of energy, the option was ruled out. The passive swell compensation was triggered by the bottom contact. A suction tube with a closed end that rested on the seabed was developed. Two high-pressure water jets helped the bottom plate to enter the seabed if necessary.

Swell compensation took place from the top of the suction pipe that was suspended from one heavy wire. The system can tolerate a vertical movement of 10 to 11m and a wave period of 7 to 9 seconds. The wire ran to the suction-tube winch by way of the cylinder with a 5.5 m turn. This system was based on pre-pressurizing the hydraulic oil in the cylinder by using a pressure vessel to bring up much of the weight of the suction tube. While the ship heaves, the pressure on the foot of the suction pipe will always remain within the acceptable values, that is, between being hoisted free from the seabed and hitting the seabed too hard. Variation of forces in the hoisting wire caused by wave action was determined by a preset pressure. Hysteresis effects of the various sheaves' friction were taken into account. The pressure on the pit ground is a function of the weight of the suction pipe. It is influenced by the mixture concentration, the pressure of jets, the acceleration forces on the intermediate pipe, and the pulling of the swell compensation wire.

Summary

Two plain suction dredgers of 3000 m^3/hr in capacity were used for the Mailiao reclamation project. The marine fill was conveyed hydraulically to the site through floating pipelines of the diameter of 700 mm directly from the borrow source. Although development of the plain suction dredgers was not exactly simple and required a lot of computing, a sufficient stable system was ultimately been created. The dredgers were ordered in June 1993 and the first vessel arrived on site in April 1994, followed by the second one in July 1994. During the 9 months period the dredger builder not only performed the design and manufacture, the development of the automatic suction pipe and the swell compensation system, but also the model tests in the wave tank. The reclamation was completed in December 1997 that was 9 months ahead of schedule. This is also a benefit from a successful cooperation between the contractor and the dredger builder.

DENSIFICATION ON RECLAIMED LANDS

The hydraulic fills are generally loose, at the relatively density ranging from 20 to 60 %. Therefore, they may experience large settlement under static or dynamic loading. In addition, countries such as Taiwan and Japan are in seismic active regions that liquefaction of reclaimed land is a real hazard. Densification measures are necessary for the improvement of reclaimed land. Methods commonly used such as dynamic compaction, vibroflotation, and resonance compaction are discussed as following

Dynamic Compaction.

The dynamic compaction procedure is usually conducted in two steps, the primary pounding and the ironing pounding. The former is performed with higher energy and wider spacing between drop points to densify deeper zones. The latter is to smoothen out the craters formed at the primary step. The primary pounding is usually divided into 3 stages. The various factors that were adopted in major Southeast Asian reclamation sites are summarized in Table 3. It shows that pounders of 230 to 250 kN in weights and dropping at the heights ranging from 15 to 25m were generally adopted for the primary step, resulting the impact energy of 300 to 600 kN-m for a single drop. Depends on the required maximum depth of improvement, the total impact energy applied for the working panels ranged from 1600 to 5000 kN-m/m^2. The maximum depth of improvement is back-calculated from the empirical formula proposed by Mayne et al (1984). The back-calculated depths of improvement in Table 3 are in close agreement with the experience elsewhere in the world. Results of the trial compactions at the Haifong site (Lo, 1998) suggested that the dynamic compaction is not effective beyond 12 meter depth. On the other hand, Yu and Hsu (1997) observed that the maximum degree of improvement occurred at the depth of about one third of the influence. At that particular depth, the degree of improvement, the ratio of increment of in-situ test value and the pre-compaction value, is ranging from 40 to 50 %.

Vibroflotation

Vibroflotation refers to a process of sand compaction through the insertion of a vibrating poker or a vibroflot into ground. The vibration provides the compaction energy that causes instant settlement due to liquefaction. In general, the vibroflot, which consists of a cylindrical body housing an electric motor and an eccentric mass, could provide the horizontal eccentric force. To assist penetration, water jets are fitted to the hose of the poker. To carry out compaction, the vibroflot is lowered to the bottom of the soil layer, with (wet process) or without the water jets (dry process), and then gradually withdrawn in 0.5 to 1.0m stages. The length of time spend at each stage depends on the soil reaction.